CHAPTER

20

Conic Section

Exercise

1. The equation of the directrix of the parabola $x^2 - 4x - 3y + 10 = 0$ is
   (a) $y = -\frac{5}{4}$ (b) $y = \frac{5}{4}$
   (c) $y = -\frac{3}{4}$ (d) $x = \frac{5}{4}$
2. The coordinates of the focus of the parabola $x^2 - 4x - 8y - 4 = 0$ are
   (a) (0, 2) (b) (2, 1)
   (c) (1, 2) (d) (– 2, – 1)
3. The parametric representation $(2 + t^2, 2t + 1)$ represents
   (a) a parabola with focus at (2, 1)
   (b) a parabola with vertex at (2, 1)
   (c) an ellipse with centre at (2, 1)
   (d) None of the above
4. If (2, 0) is the vertex and Y-axis is the directrix of a parabola, then its focus is
   (a) (2, 0) (b) (– 2, 0)
   (c) (4, 0) (d) (– 4, 0)
5. If the line $x + y - 1 = 0$ touches the parabola $y^2 = kx$, then the value of k is
   (a) 4 (b) – 4
   (c) 2 (d) – 2
6. If 2, 3 and – 2 be the ordinate of vertices of triangle inscribed on a parabola $y^2 = 16x$ then area of triangle is
   (a) 5 / 8 sq unit (b) 3 / 5 sq unit
   (c) 3 / 8 sq unit (d) 8 / 5 sq unit
7. Find the equation of the parabola whose vertex and focus are on the X-axis at distance a and a' represented from the origin.
   (a) $y^2 = 4(a' + a)(x - a)$ (b) $y^2 = 4(a' + a)(x + a)$
   (c) $y^2 = 4(a' - a)(x - a)$ (d) $y^2 = 4(a' - a)(x + a)$
8. The parabola $(y - k)^2 = 4p(x - h)$, where $p > 0$ opens to the
   (a) right (b) left
   (c) upward (d) downward
9. The line $2x + y + \lambda = 0$ is a normal to the parabola $y^2 = -8x$, then λ is
   (a) 12 (b) – 12
   (c) 24 (d) – 24
10. If straight line $y - x - 2 = 0$ is the tangent of the parabola $y^2 = 8x$ then find the point of contact.
    (a) (4, 2) (b) (2, – 4)
    (c) (2, 4) (d) (– 2, 4)
11. The line $lx + my + n = 0$ a normal to the parabola $y^2 = 4ax$ if
    (a) $al(l^2 + 2m^2) + m^2n = 0$
    (b) $al(l^2 + 2m^2) = m^2n$
    (c) $al(2l^2 + m^2) = -m^2n$
    (d) $al(2l^2 + m^2) = 2m^2n$
12. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabola $y^2 = 4ax$ and $x^2 = 4ay$, then
    (a) $d^2 + (2b + 3c)^2 = 0$ (b) $d^2 + (3b + 2c)^2 = 0$
    (c) $d^2 + (2b - 3c)^2 = 0$ (d) $d^2 + (3b - 2c)^2 = 0$
13. If the parabolas $y^2 = 4x$ and $x^2 = 32y$ intersect at (16, 8) at an angle θ, then θ is equal to
    (a) $\tan^{-1}\left(\frac{3}{5}\right)$ (b) $\tan^{-1}\left(\frac{4}{5}\right)$
    (c) π (d) $\frac{\pi}{2}$
14. What is the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of the latus rectum?
    (a) 9 sq units (b) 12 sq units
    (c) 14 sq units (d) 18 sq units

15. The curve represented by $x = 3(\cos t + \sin t)$, $y = 4(\cos t - \sin t)$ is
(a) ellipse (b) parabola
(c) hyperbola (d) circle

16. The equation of the ellipse which passes through the point (– 3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is
(a) $3x^2 + 6y^2 = 33$
(b) $5x^2 + 3y^2 = 48$
(c) $3x^2 + 5y^2 = 32$
(d) None of these

17. If the ellipse $9x^2 + 16y^2 = 144$ intercepts the line $3x + 4y = 12$, then what is the length of the chord so formed?
(a) 5 unit (b) 6 unit
(c) 8 unit (d) 10 unit

18. The eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose latus rectum is half of its minor axis is
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{\frac{2}{3}}$
(c) $\frac{\sqrt{3}}{2}$ (d) None of these

19. The equation $\frac{x^2}{10-a} + \frac{y^2}{4-a} = 1$ represents an ellipse if
(a) $a < 4$ (b) $a > 4$
(c) $4 < a < 10$ (d) $a > 10$

20. The eccentricity of the conic $x^2 - 4x + 4y^2 = 12$ is
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$
(c) $\sqrt{3}$ (d) None of these

ANSWERS

1.	(b)	**2.**	(b)	**3.**	(b)	**4.**	(c)	**5.**	(b)	**6.**	(a)	**7.**	(c)	**8.**	(a)	**9.**	(c)	**10.**	(c)
11.	(a)	**12.**	(a)	**13.**	(a)	**14.**	(d)	**15.**	(a)	**16.**	(c)	**17.**	(a)	**18.**	(c)	**19.**	(a)	**20.**	(a)

Explanations

1. (b) Equation $x^2 - 4x - 3y + 10 = 0$ can be written as $(x-2)^2 = 3(y-2)$
Here, origin (0, 0) is shifted at (2, 2)
So, the equation can be written as $X^2 = 3Y$
where $x = X + 2$ and $y = Y + 2$
The equation of directrix of above parabola is
$Y = -a$ where $a = \frac{3}{4}$
$\Rightarrow \quad y - 2 = -\frac{3}{4}$
$\Rightarrow \quad y = 2 - \frac{3}{4} = \frac{5}{4}$

2. (c) Parabola $x^2 - 4x - 8y - 4 = 0$ can be written as $(x-2)^2 = 8(y+1)$ and can be written as $X^2 = 8Y$
Here, origin is shifted to (2, – 1) and $a = 2$
Focus is given by $(0, a) = (0,2)$
i.e., $X = 0$ and $Y = 2$
$\Rightarrow x - 2 = 0$ and $y + 1 = 0$
$\Rightarrow x = 2$ and $y = 1$
Hence, focus = (2,1)

3. (b) Given, $x = (2 + t^2)$ and $y = (2t + 1)$
$\Rightarrow x - 2 = t^2$ and $\frac{y-1}{2} = t$
$\Rightarrow x - 2 = \frac{(y-1)^2}{4}$
$\Rightarrow (y-1)^2 = 4(x-2)$
This is the equation of a parabola whose vertex is (2, 1).

4. (c) Since, $A(2, 0)$ is the vertex and Y axis is the directrix.
So, axis of parabola is X axis.
Let the focus is $S(a, 0)$
Then, vertex $A(2, 0)$ is the mid-point of $S(a, 0)$ and (0, 0).
$\Rightarrow \frac{a+0}{2} = 2 \Rightarrow a = 4$
Hence, focus = (4, 0)

5. (b) Line $y = x - 1$ touches the parabola $y^2 = kx$
$\Rightarrow (x-1)^2 = kx \Rightarrow x^2 - x(2+k) + 1 = 0$
This equation has equal roots.
So, $(2+k)^2 = 4(1) \Rightarrow k = -4$

6. (a) Given points on the parabola $y^2 = 16x$ have ordinates 2, 3 and – 2.

So, points are $\left(\frac{1}{4}, 2\right), \left(\frac{9}{16}, 3\right)$ and $\left(\frac{1}{4}, -2\right)$

∴ Area of triangle having these vertices is

$$= \frac{1}{2}\begin{vmatrix} 1/4 & 2 & 1 \\ 9/6 & 3 & 1 \\ 1/4 & -2 & 1 \end{vmatrix} = \frac{5}{8} \text{sq unit}$$

7. (c) Given vertex and focus of the parabola are $A(a,0)$ and $S(a', 0)$. Let $M(x, 0)$ be any point on the directrix on the axis of the parabola.

Then, A is the middle point of SM.

$\Rightarrow \frac{x + a'}{2} = a \Rightarrow x = 2a - a'$

$\Rightarrow$ Equation of the directrix is $x - (2a - a') = 0$

For parabola $PS^2 = PM^2$

$\Rightarrow (x - a')^2 + (y - 0)^2 = \{x - (2a - a')\}^2$

$\Rightarrow x^2 + a'^2 - 2a'x + y^2$

$= x^2 + 4a^2 + a'^2 - 4aa' - 2x(2a - a')$

$\Rightarrow y^2 = 4\{a^2 - aa' - ax + a'x\}$

$\Rightarrow y^2 = 4(a' - a)(x - a)$

8. (a) Parabola $(y - k)^2 = 4p(x - h)$, where $p > 0$ opens to the right.

9. (c) ∵ Line $2x + y + \lambda = 0$ *i.e.*, $y = -2x - \lambda$ is a normal to the parabola $y^2 = -8x$

$\Rightarrow \lambda = -2(2)(-2) - (2)(-2)^3$

$\Rightarrow \lambda = 24$

10. (c) Line $y - x - 2 = 0$ *i.e.*, $y = x + 2$ is the tangent of the parabola $y^2 = 8x$

$\Rightarrow (x + 2)^2 = 8x$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow x = 2$

So, from line $y = x + 2 = 2 + 2 = 4$

Hence, point of contact = (2, 4)

11. (a) Line $lx + my + n = 0$ *i.e.*, $y = -\frac{l}{m}x - \frac{n}{m}$ is a normal of the parabola $y^2 = 4ax$

$\Rightarrow -\frac{n}{m} = \frac{2al}{m} + \frac{2al^3}{m^3}$

[if $y = Mx + C$ is normal to $y^2 = 4ax$ then $C = -2aM - aM^3$]

$\Rightarrow -nm^2 = 2alm^2 + al^3$

$\Rightarrow al(l^2 + 2m^2) + m^2n = 0$

12. (a) Intersection points of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ are (0, 0) and $(4a, 4a)$.

These points lie on the line $abx + 3cy + 4d = 0$

$\Rightarrow d = 0$ and $2b + 3c = 0$

$\Rightarrow d^2 + (2b + 3c)^2 = 0$

13. (c) Slope of tangent at (16, 8) for the parabola $y^2 = 4x$

is $m_1 = \left(\frac{dy}{dx}\right)_{(16,8)} = \left(\frac{4}{2y}\right)_{(16,8)} = \frac{2}{8} = \frac{1}{4}$

Slope of tangent at (16, 8) for the parabola $x^2 = 32y$

is $m_2 = \left(\frac{dy}{dx}\right)_{(16,8)} = \left(\frac{2x}{32}\right)_{(16,8)} = 1$

Angle between both the tangents is given by

$\tan\theta = \frac{m_2 - m_1}{1 + m_1m_2} = \frac{1 - 1/4}{1 + 1/4}$

So, $\theta = \tan^{-1}\left(\frac{3}{5}\right)$

14. (d) For parabola $x^2 = 12y$, $a = 3$

Extremities of the latus rectum are $(\pm 2a, a)$

i.e., (± 6, 3) and vertex is (0, 0)

Area of the triangle $\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 3 & 1 \\ -6 & 3 & 1 \end{vmatrix} = 18$ sq units.

15. (a) Given curve is

$x = 3(\cos t + \sin t)$ and $y = 4(\cos t - \sin t)$

$\Rightarrow \frac{x^2}{9} = 1 + \sin 2t$ and $\frac{y^2}{16} = 1 - \sin 2t$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 2$

or $\frac{x^2}{18} + \frac{y^2}{32} = 1$

This is the equation of an ellipse.

16. (c) Let the equation of the ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

It passes through (– 3, 1)

So, $\frac{9}{a^2} + \frac{1}{b^2} = 1 \Rightarrow 9b^2 + a^2 = a^2b^2$

$\Rightarrow 9a^2(1 - e^2) + a^2 = a^2 \times a^2(1 - e^2)$

$\{\because b^2 = a^2(1 - e^2)\}$

$\Rightarrow 9 - 9e^2 + 1 = a^2(1 - e^2)$

$\Rightarrow 10 - 9 \times \frac{2}{5} = a^2\left(1 - \frac{2}{5}\right)$ $\left\{\because e = \sqrt{\frac{2}{5}}\right\}$

$\Rightarrow a^2 = \frac{32}{3}$

So, $b^2 = a^2(1 - e^2) = \frac{32}{3}\left(1 - \frac{2}{5}\right) = \frac{32}{5}$

Hence equation of ellipse is $\frac{x^2}{32/3} + \frac{y^2}{32/5} = 1$

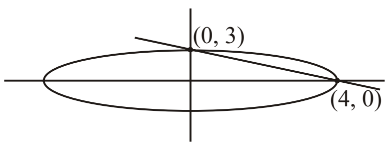
or $3x^2 + 5y^2 = 32$.

17. (a) Ellipse $9x^2 + 16y^2 = 144$ can be written as

$$\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$$

and line $3x + 4y = 12$ can be written as $\frac{x}{4}+\frac{y}{3}=1$

So, line intercepts the ellipse at (0, 3) and (4, 0)



Therefore, length of chord = ditance between (0, 3) and (4, 0) = $\sqrt{4^2+3^2}=5$ unit

18. (c) Given, latus rectum = $\frac{1}{2}$(minor axis)

$$\Rightarrow \frac{2b^2}{a}=\frac{1}{2}(2b)$$

$$\Rightarrow 2b^2 = ab \Rightarrow \frac{b}{a}=\frac{1}{2}$$

$$\because e=\sqrt{1-\frac{b^2}{a^2}}$$

$$\Rightarrow e=\frac{\sqrt{3}}{2}$$

19. (a) Given $\frac{x^2}{10-a}+\frac{y^2}{4-a}=1$ represents an ellipse

So, $10 - a > 0$ and $4 - a > 0$

$\Rightarrow$ $10 > a$ and $4 > a$

Hence, $a < 4$

20. (a) Given equation $x^2 - 4x + 4y^2 = 12$ can be written as $(x - 2)^2 + 4(y - 0)^2 = 8$

$$\Rightarrow \frac{(x-2)^2}{8}+\frac{(y-0)^2}{2}=1$$

Here, $a^2 = 8$ and $b^2 = 2$

$\because b^2 < a^2$

So, eccentricity $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{2}{8}}=\frac{\sqrt{3}}{2}$